Supersonic Turbulent Boundary Layer in the Symmetry Plane of a Cone at Incidence

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Nomenclature

i = angle of attack
M = Mach number
M' = pressure gradient parameter $\dfrac{1}{u_e \sin\theta_c}\dfrac{\partial w_e}{\partial \phi}$
u, v, w = velocity components along the x, y and ϕ directions
x = coordinate along generators of cone
y = coordinate normal to surface
δ = boundary-layer thickness
θ_c = cone half-angle
ϕ = azimuth angle

Theme

TWO approximate solutions for the case of zero incidence, each corresponding to one of two kinds of similarity variables generally used, are discussed by comparing them to an exact solution obtained by a finite difference method. For the cone at incidence, the equations are solved in the symmetry plane for a wide range of Reynolds numbers. The influence of this last parameter on the longitudinal and azimuthal characteristic properties of the boundary layer is emphasized.

Contents

Locally similar solutions have been often used by many authors for the bidimensional or axisymmetric turbulent boundary-layer problem. More recently Adams[1] has used such solutions for the three-dimensional supersonic boundary layer around a cone at incidence. It is demonstrated in the present paper that some cautions should be taken when using such approximate solutions. Initially the choice of the similarity variable in the zero incidence case will be discussed. Then the abscissa effect on the main physical quantities of the boundary layer will be emphasized with and without incidence, for the particular case of the symmetry plane. All the calculated results given in the present paper are obtained for a 9° half-angle cone which has a surface temperature of 300K and which is placed in a supersonic flow at $M_\infty = 7$. The stagnation temperature is 600K and the freestream Reynolds number, Re_∞, is 1.10×10^5 per cm.

The turbulent boundary-layer equations on a cone at incidence have been derived from the Navier-Stokes equations in a previous paper.[2] For the zero approximation and for a two-layer eddy-viscosity model, one obtains[2] a system identical to the one derived by Adams.[1]

In the zero incidence case, the Lees-Dorodnitsyn variable which is usually employed in the laminar case has been first used as in Ref. 1. This permits a great simplification of the equations. The corresponding approximate solutions, calculated in the zero incidence case by a shooting method, have been compared with the exact solution obtained with a finite difference method. This comparison reveals two main features. The flow at the wall which is characterized by both the skin friction coefficient, C_f, and the heat transfer coefficient, C_h appears to be well predicted when using this similarity variable. But the flow at the external edge, characterized by the boundary-layer thicknesses, is not so well predicted. This last feature can be explained. Indeed, the use of the similarity assumption implies a relation between C_f and the momentum thickness, δ_2, which depends on the similarity variable used and which always differs from the exact turbulent equation of von Kármán. The relation, implied by the momentum equation which is transformed with the Lees-Dorodnitsyn variable, has been shown to coincide with the laminar equation of Kármán. For the usual turbulent similarity variable, y/δ, the following relation was derived.

$$C_f = (1+m)\,\delta_2/x \quad \text{with} \quad m = x/\delta \cdot \mathrm{d}\delta/\mathrm{d}x$$

Probably because of the conical nature of the external flow, this is very close to the turbulent equation of von Kármán. The solution which is also calculated with a shooting method confirms this relation and appears to agree very well with the exact solution everywhere in the boundary layer. The variation of m, that appears in the equations as a parameter to be adjusted by an iterative process, is given in terms of Re_x. This variation is shown to be very much like the one usually described in the flat plate case. In order to make a more accurate comparison of such a behavior between the flat plate case and the cone case, the relations C_f, δ_2/x, C_h are derived in powers of the Re_x.

For the symmetry plane of the cone at incidence (excluding the case where the azimuthal derivative of the velocity takes infinite values), the governing equations can be reduced to a set of first-order differential equations on applying the similarity assumption. In addition to the parameters appearing in the zero incidence case, these equations which are obtained by using the turbulent similarity variable contain the pressure gradient parameter, M', that characterizes the crossflow due to the incidence. There is no particular problem in solving these equations for the

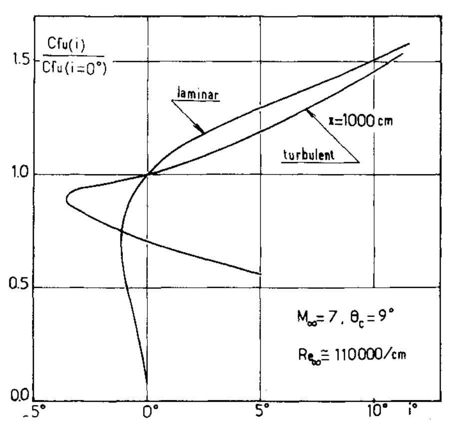

Fig. 1 Variation of the skin friction coefficient vs incidence.

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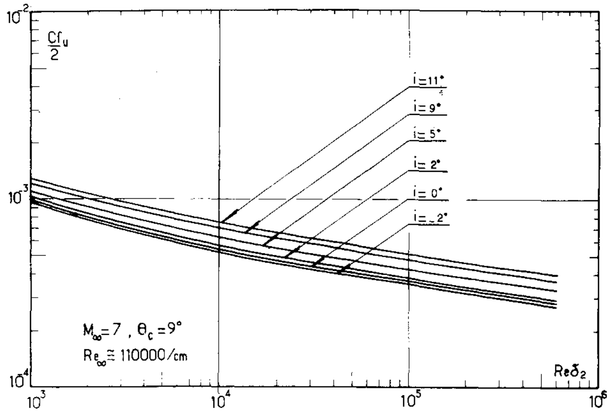

Fig. 2 Longitudinal distribution of the skin friction coefficient.

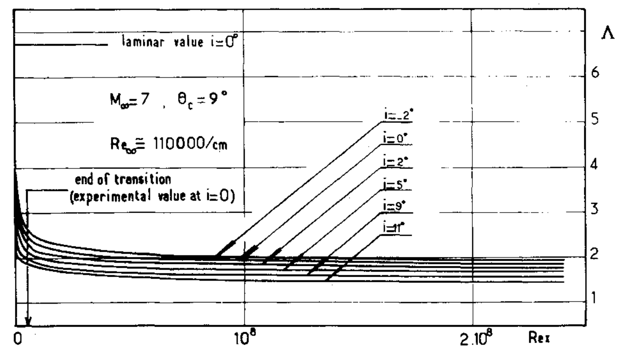

Fig. 3 Longitudinal variation of the limiting streamlines deviation.

windward plane where $M' > 0$. But in the leeward plane where $M' < 0$, there is, as in the laminar case, a critical incidence, i_{cr}, beyond which the shooting method is no longer convergent. The value of i_{cr} depends upon Re_x in the turbulent case and appears to be always much greater than the laminar value (Fig. 1).

In Fig. 2 the longitudinal skin friction coefficient, C_{fu}, for some values of incidence up to 11°, together with results of the zero incidence case, is presented in terms of $Re\,\delta_2$. All these curves have the same qualitative behavior and involve about the same exponent for their longitudinal variation which is expressed locally in powers of Re_x. This remark again is true for the momentum thickness of the boundary layer and for the coefficient of heat transfer. The powers s_{01}, s_{02} and s_{03} of the variation of C_{fu}, δ_2/x and C_h, respectively, in terms of Re_x have been calculated and compared with the corresponding zero incidence values. The incidence appears to have no significant effect on the variation (in terms of Re_x) of the longitudinal quantities of the boundary layer.

Concerning the three-dimensional quantities we have particularly emphasized the deviation of the limiting streamlines at the wall and near the plane of symmetry. This deviation is characterized by $(\partial\alpha_p/\sin\theta_c\,\partial\phi)_{\phi=0,\pi}$ where α_p, defined from the ratio

$$\frac{w}{u} = \frac{(\partial w/\partial y)_{y=0}}{(\partial u/\partial y)_{y=0}} = tg\alpha_p$$

denotes the angle of the limiting streamlines with the generators. When the incidence goes to zero, α_p and M' vanish together, whereas the ratio

$$\Lambda = \frac{1}{M'}\left(\frac{1}{\sin\theta_c}\frac{\partial\alpha_p}{\partial\phi}\right)_{\phi=0,\pi}$$

is well defined at $i = 0$. This ratio has been calculated using its derivative with respect to ϕ instead of the azimuthal momentum equation. The results are given in Fig. 3 in terms of Re_x. Those results of the experimental study of Rainbird,[3] where α_p was claimed to be independent of x, appear to be surprising. However this difference can be partly explained. The steep increment of Λ, as Re_x goes to zero, is a fictitious effect caused by the eddy viscosity. That is, when $Re_x \to 0$, the eddy viscosity, μ_t, used in the present study vanishes. Therefore the governing system approaches continuously the corresponding laminar boundary-layer equations, for which Λ, no longer depending upon x, has a value much larger than the turbulent one. On the other hand, the results given by Rainbird[3] in the case where $\theta_c = 12.5°$, $i/\theta_c = 1.8$, $M_e = 1.134$ and $T_w = 1.257$ relate to a relatively small range of values of x downstream of the transition. Here Re_x covers the range of 6.2×10^6 to 2.1×10^7. Now for this range, the theoretical variation of Λ which depends upon Re_x appears to be of the same order as the measuring accuracy of the experimental results.

In this way, these results cannot contradict the theoretical variation of Λ in terms of x. However, this variation becomes less and less important when $Re_x \to \infty$. That is, this effect would be considerable only when the boundary layer is transitional or turbulent with Reynolds number not too large. Furthermore, it cannot be concluded that the observed theoretical variation of Λ is realistic when Re_x is slightly greater than the transition Reynolds number. Recalling that the three-dimensional eddy-viscosity model is empirically deduced from the two-dimensional one, it can only be said at this time that additional experiments can well formulate the theory.

References

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